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e+e- Plasma Photon Source

E. P. Hartouni

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Ed Hartouni

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This note addresses the idea of a photon source that is based on an e^+e^- plasma created by co-propagating beams of e^+ and e^-. The plasma has a well defined temperature, and the thermal distribution of the charged particles is used to average over the relative-velocity cross section multiplied by the relative-velocity. Two relevant cross sections are the direct "free-free" annihilation of e^+e^- pairs in the plasma, and the radiative recombination of e^+e^- pairs into positronium (Ps) which subsequently undergoes annihilation.

Photon energy

The photons of interest have an energy of 0.511 MeV in the reference frame of the plasma. The co-propagating beams of kinetic energy T_b have energy $E_b = T_b + m_e c^2$ and momentum $p_b = \sqrt{T_b(T_b + 2m_e c^2)}$. Both beams have the same energy and momentum (neglecting the thermal motion). The energy of the subsequent radiation, "boosted" into the lab reference frame uses the quantities:

$$\begin{aligned}
\beta &= \frac{2p_b}{2E_b} = \frac{\sqrt{T_b(T_b + 2m_e c^2)}}{T_b + m_e c^2} \approx \frac{\sqrt{1 + 2m_e c^2/T_b}}{1 + m_e c^2/T_b} \\
\gamma &= \frac{T_b + m_e c^2}{m_e c^2} = \frac{T_b}{m_e c^2} + 1
\end{aligned}$$

The energy of a photon moving in the direction of the co-propagating beam and emitted at an angle θ^* in the lab frame is then given by:

$$E_\gamma = \gamma m_e c^2 \left(1 + \beta \cos\theta^*\right) \tag{1}$$

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to make the beam energy explicit, we right this:

$$E_\gamma = \left(T_b + m_e c^2\right) \times (1 + \beta \cos\theta^*) \tag{2}$$

The energy of the photon emitted in the beam direction is "set" by the beam kinetic energy T_b. Where $m_e c^2 = 0.511$ MeV, and $\beta \approx 1$, an annihilation photon emitted with $\theta^* = 0$ (along the direction of the co-propagating beams) will have an energy of 20 MeV when $T_b \approx 9.5$ MeV. Changing the beam energy changes the photon energy allowing for the photon source to have variable energy output.

The energy of the photon E_γ is a function of the angle of decay in the rest frame of the plasma, $\cos\theta^*$. The photon distribution in the rest frame is given by:

$$\frac{dN_\gamma}{d\cos\theta^*} = \text{constant} \tag{3}$$

and noting that:

$$\frac{dN_\gamma}{dE_\gamma} = \frac{dN_\gamma}{d\cos\theta^*}\frac{d\cos\theta^*}{dE} \tag{4}$$

and given $E_\gamma = \gamma m_e c^2 (1 + \beta\cos\theta^*)$

$$\frac{dN_\gamma}{dE_\gamma} = \frac{dN_\gamma}{d\cos\theta^*}\frac{1}{\gamma\beta M_e} = \frac{\text{constant}}{\gamma\beta M_e} \tag{5}$$

The energy spectrum is "flat." A band of energies ΔE_γ is selected by collimating the beam, restricting the decay angles in the lab frame:

$$\frac{dN_\gamma}{d\cos\theta} = \frac{dN_\gamma}{d\cos\theta^*}\frac{d\cos\theta^*}{d\cos\theta} = \text{constant}\frac{d\cos\theta^*}{d\cos\theta} \tag{6}$$

The relativistic transformations give:

$$\frac{d\cos\theta^*}{d\cos\theta} = \frac{1}{\gamma^2\left(\beta\cos\theta - 1\right)^2} \tag{7}$$

and then

$$\frac{dN_\gamma}{d\cos\theta} = \frac{\text{constant}}{\gamma^2\left(\beta\cos\theta - 1\right)^2} \tag{8}$$

a very forward peaked distribution at high beam energies ($\gamma \approx 20$ in the above example). Figure 1 shows the angular distribution of photons from two co-propagating 10 MeV beams. The photon energy is a function of the

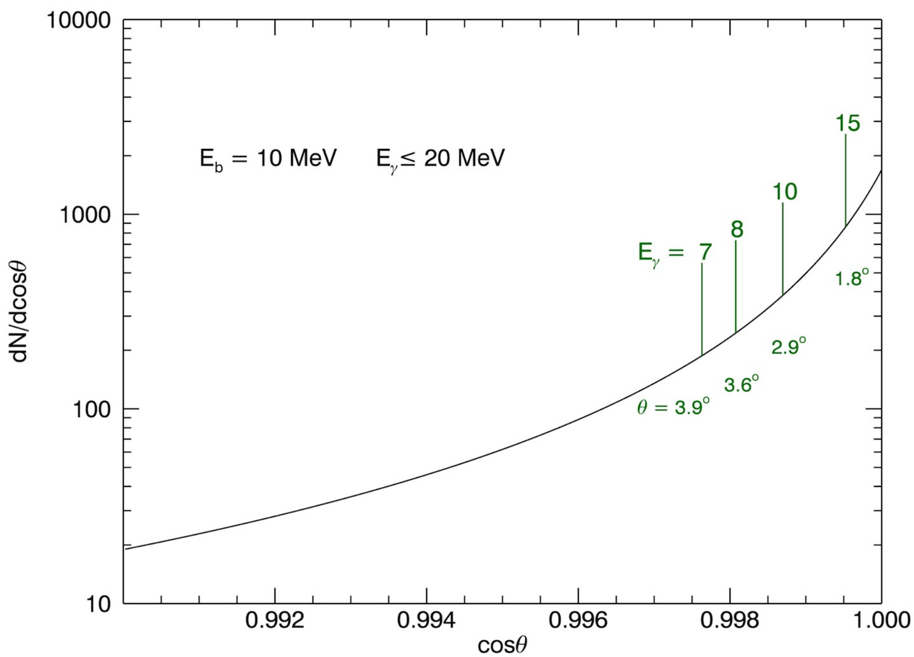

Figure 1: Photon angular distribution of a 10 MeV electron and positron beam energy. Photon energy is indicated by green "ticks."

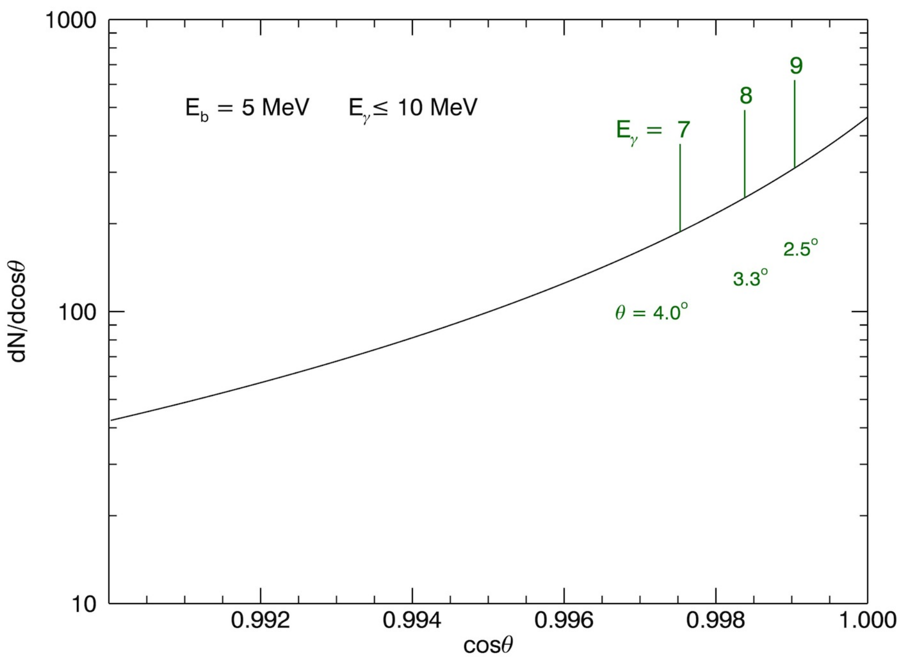

Figure 2: Photon angular distribution of a 5 MeV electron and positron beam energy. Photon energy is indicated by green “ticks.”

emission angle in the lab relative to the beam direction. The angles for specific energies are also indicated. In this example, a collimator which cuts off angles $\theta \leq 3.9°$ would accept photons of energies ≥ 7 MeV. The total fraction of photons making it through the collimator is 59%. For the beam energy case of 5 MeV, shown in Figure 2, a similar collimator angle also cuts at 7 MeV, but the total fraction of photons making it through is 30%.

The same photon acceptance fraction is the $\Delta E_\gamma / E_\gamma$. In this source concept, the "bandwidth" of the beam energy can be selected by the acceptance angle of the collimator. The number of photons that are pass through the collimator is that same fraction. For smaller $\Delta E_\gamma / E_\gamma$ there is a lower flux at fixed photon production.

This suggests an operating mode where beam intensity is set by the electron circulating current in the storage ring, the high energy endpoint of the photons is set by the beam energies, being roughly twice that energy, and the low energy endpoint set by collimating photon beam which cuts off the beam at a lab angle corresponding to the desired beam lowest energy.

Rate of photon production

The number of annihilations in the center-of-mass frame of the e^+e^- beams is given by the rather general expression:

$$R = n_{e^+} n_{e^-} < \sigma v > V \tag{9}$$

where the variables n are the densities of particles, $< \sigma v >$ is the relevant cross section multiplied by the pair relative velocity averaged over the relative-velocity distributions, and V is the interaction volume. This expression can be rewritten:

$$R = N_{e^+} N_{e^-} < \sigma v > \frac{1}{A\ell} \tag{10}$$

Where the N variables are the total number of particles in the interaction volume defined by the product of the beam area, A and the length of the interaction volume ℓ (in the center-of-mass reference frame).

To calculate $< \sigma v >$ we make use of the expression:

$$< f > = \frac{\int_{-1}^{1} d\cos\theta \int_0^\infty dv v^2 \int_0^\infty dV V^2 f e^{-aV^2 - bv^2 - 2dVv\cos\theta}}{\int_{-1}^{1} d\cos\theta \int_0^\infty dv v^2 \int_0^\infty dV V^2 e^{-aV^2 - bv^2 - 2dVv\cos\theta}} \tag{11}$$

where V is the center-of-mass velocity, v is the relative velocity and $\cos\theta$ is the angle between the two, the function $f = f(v, V, \cos\theta)$ and

$$a = \frac{m_e}{2}\left(\frac{1}{\theta_+} + \frac{1}{\theta_-}\right) \tag{12}$$

$$b = \frac{m_e}{8}\left(\frac{1}{\theta_+} + \frac{1}{\theta_-}\right) \tag{13}$$

$$d = \frac{m_e}{4}\left(\frac{1}{\theta_+} - \frac{1}{\theta_-}\right) \tag{14}$$

where m_e is the electron mass and $\theta_\pm = kT_\pm$ the "temperature" of the electron and positron (as designated by the subscript).

$$\int_{-1}^{1} d\cos\theta \int_0^\infty dv v^2 \int_0^\infty dV V^2 e^{-aV^2 - bv^2 - 2dVv\cos\theta} = \pi\left(\frac{\theta_+\theta_-}{m_e^2}\right)^{3/2} \tag{15}$$

Note that in what follows the relevant beam temperatures are of order 1 eV and less, which makes this problem non-relativistic in the center-of-mass. The two relevant cross sections are the direct e^+e^- pair annihilation and the radiative-recombination cross section in which positronium (Ps) is created by radiative recombination to an excited state, the state cascades to lower energy states and the e^+e^- pair annihilates along the way.

The "free" e^+e^- pair annihilation cross section is given by:

$$\sigma_{e^+e^-\to\gamma\gamma}(v) = \left[\frac{\pi r_0^2}{2}\left(\frac{c}{v}\right)\right]\frac{2\pi\alpha c/v}{1 - e^{-2\pi\alpha c/v}} \tag{16}$$

where the bracketed term is the "bare" cross-section, and the other term the "Coulomb correction," both of these being the non-relativistic expressions in the pair relative velocities v. r_0 is the classical radius of the electron 2.82×10^{-15} m and α is the fine-structure constant.

For temperatures lower than 30 eV this can be approximated as:

$$\sigma_{e^+e^-\to\gamma\gamma}(v) \approx \pi^2 r_0^2 \alpha \left(\frac{c}{v}\right)^2 \tag{17}$$

resulting in:

$$<\sigma v> \approx \sqrt{\frac{2\pi m_e c^2}{\theta_+ + \theta_-}}\alpha\pi r_0^2 c \tag{18}$$

with the rate from Eq. 10 being:

$$R_{e^+e^-\to\gamma\gamma} = \frac{N_{e^+}N_{e^-}}{V}\sqrt{\frac{2\pi m_e c^2}{\theta_+ + \theta_-}}\alpha\pi r_0^2 c \tag{19}$$

For $\theta \approx 1$ eV we have:

$$R_{e^+e^-\to\gamma\gamma} = 7\times 10^{-14}[\mathrm{cm}^3/\mathrm{s}]\frac{N_{e^+}N_{e^-}}{V} \tag{20}$$

If the beam cross sections are of order 0.1 cm and the interaction region length 100 cm, and there are 10^7 electrons and 10^7 positrons in the interaction volume, there will be ≈ 1 e^+e^- pair annihilation per crossing. However, the interaction region length would be 1000 cm in the lab frame (due to the relativistic dilation), a 100 cm *lab* frame interaction region length corresponds to a 10 cm center-of-mass frame interaction region length, reducing the γ-ray production rate by the same factor. In general:

$$\ell = \ell_{\mathrm{lab}}/\gamma = \ell_{\mathrm{lab}} m_e c^2/E_{\mathrm{beam}} \tag{21}$$

If we assume that the storage-ring has dimension of roughly 4 times the lab interaction region length (take this to be 1 meter) and one circulating bunch, then the period of bunch crossings is roughly 10 ns and the γ-ray production rate is 0.1 γ/crossing $\times 1$ crossing/10 ns = 10^7 γ/s.

The currents represented by 10^7 particles in the interaction volume are 10 mA, this is small for e^- beams and large for e^+ beams. Setting the e^- beam current to 1A would require an e^+ beam to be 100 μA (the product remaining the same). This is probably a stretch given existing technologies.

The treatment of the e^+e^- pair radiative recombination into positronium (Ps) and it's subsequent radiative decay to the ground state and the possibility of annihilation is treated in R. J. Gould, *The Astrophysical Journal*, **344**, 232 (1989), "Direct Positron Annihilation and Positronium Formation in Thermal Plasmas." The recombination rate (for a thermalized e^+e^- pair plasma at temperature T) is given by Gould's Eq. (7):

$$R_{e^+e^-\to Ps} = N_{e^+}N_{e^-} < \sigma_r v > \frac{1}{V} \tag{22}$$

and we have from his Eqs. (9) and (10):

$$< \sigma_r v >= 4C\sqrt{\frac{2\theta}{\pi m_e}} y\phi(y)\bar{g}(y) \tag{23}$$

where $\theta = 2kT$, $C = 2^6 3^{-3/2} \alpha^3 \pi a_0^2$, $y = \mathrm{Ry}/\theta$. The functions ϕ and $\bar{g}$ are given in various tables and formulas (as an *ansatz* replace θ by $\theta_+ + \theta_-$ where the temperatures of the electrons and positrons differ). Ry is the Rydberg energy (13.6 eV) and $a_0 = 5.292 \times 10^{-9}$ cm is the Bohr radius. For this application, once the e^+e^- pair capture into Ps the total cascade and annihilation lifetimes have to be short compared to the interaction region co-propagation times of order 1 - 10 ns. The para-Ps state lifetime to 2 photons is 1.25×10^{-10} s, we infer that all of these transitions would result in γ-ray production. While the ortho-Ps states decaying to 3 photons has a lifetime of 1.33×10^{-7} s, these Ps states will exit the interaction region before making the transition. For practical estimates, only the $n = 1$ need be considered.

The function $\phi(y)$ is given as:

$$\phi(y) = \frac{1}{2}\left[1.735 + \ln y + (6y)^{-1}\right] \tag{24}$$

for $y \geq 1$. This the domain of interest (generally not interested above $\theta \approx 30$ eV, which corresponds to $y = 0.45$ for which Eq. 24 is still a good approximation). The averaged Gaunt factors appear in Table 2 of Gould's paper: The radiative capture rates are about an order of magnitude more for $kT < 1$

T (10^4 K)	θ (eV)	y	$\bar{g}$
0.25	0.21	65.28	0.893
0.5	0.42	32.64	0.891
1	0.83	16.32	0.890
2	1.67	8.16	0.882
4	3.33	4.08	0.881
8	6.67	2.04	0.881
16	13.33	1.02	0.887
32	26.67	0.51	0.894
64	53.33	0.26	0.913
128	106.67	0.13	0.925
256	213.33	0.06	0.928
512	426.67	0.03	0.918

Table 1: Averaged recombination Gaunt factors from Gould's Table 2.

eV where roughly 90% of the captures are in the ground state and result in the annihilation of the e^+e^- pair into 2 γ-rays. Only one of these γ-rays are used.

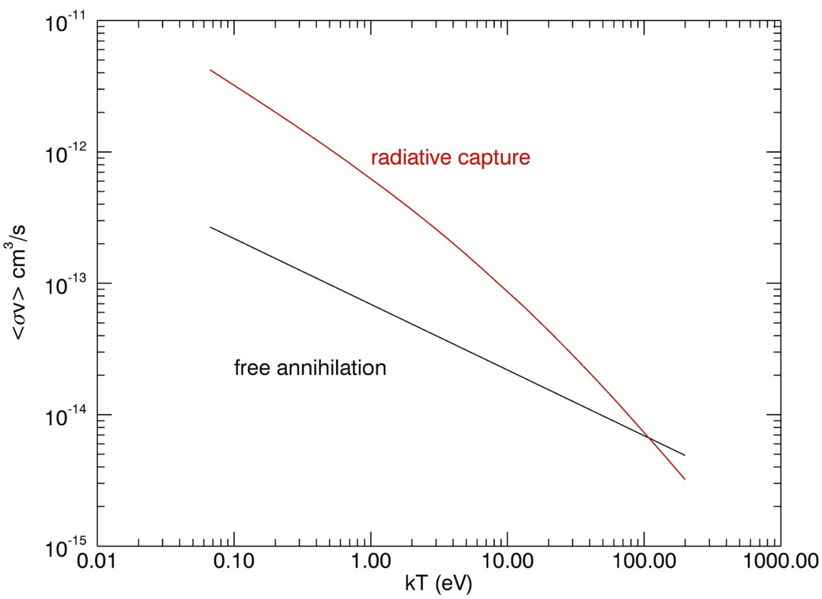

Figure 3: Reaction rates for e^+e^- pair free annihilation into 2γ and for radiative recombination into positronium which subsequently cascades to the ground state and annihilates.

Rate estimate

The beam current is related to the number of beam particles by:

$$N = \frac{I \Delta t}{e} \tag{25}$$

where I is the circulating current (in Amps) $e = 1.6 \times 10^{-19}$ Coulombs the charge of the electron, N is the number of particles and Δt is the time of interest (in seconds, here it is the overlap time of the beams).

This rate in the plasma frame is written:

$$R_{\text{plasma}} = \frac{N_+ N_-}{A\ell} < \sigma v > r \tag{26}$$

Where A is the area of the co-propagating beams, ℓ is the length of the co-propagating region in the beam frame and r is the ratio of the orbit that the beams are co-propagating. In the lab frame this is:

$$r = \frac{L}{C} \tag{27}$$

where L is the length of the co-propagating beam overlap and C is the circumference of the beam in the lab frame. This ratio is independent of the frames.

To go to the lab frame we need to recognize that:

$$\begin{aligned} \ell &\rightarrow L\gamma \\ R_{\text{plasma}} &\rightarrow R_{\text{lab}}\gamma \end{aligned}$$

Where L is the length of the beam overlap region in the lab frame, and γ the relativistic boost. So, writing in terms of lab frame quantities:

$$R_{\text{lab}} = \frac{1}{\gamma^2} \frac{N_+ N_-}{AC} < \sigma v > \tag{28}$$

where the temperatures characterizing $< \sigma v >$ are in the plasma reference frame.

Written in terms of the beam currents:

$$R_{\text{lab}} = \frac{1}{\gamma^2} \frac{I_+ I_-}{AC} \frac{\Delta t^2}{e^2} < \sigma v > \tag{29}$$

Order of magnitude values for a 20 MeV high energy endpoint might be: $\gamma = 10$, $I_- = 10^{-1}$A, $I_+ = 10^{-4}$A, $\Delta t = 10^{-8}$ s, $A = 10^{-3}$ cm^2, $C = 400$ cm and $< \sigma v >$(1eV)$= 10^{-12}$ cm^3/s yields the value: $R_{\text{lab}} \approx 10^3$ γ/s. Not included in this estimate is the fraction of Ps states that decay into a useful γ (the Ps lifetimes are dilated by the boost).

Concluding thoughts

The assumed accelerator technology is rather conservative. The beam emittance and temperatures that could possibly be obtained have not been explored in this calculation. Asymmetric beam temperatures have been invoked in proposed designs for a similar anti-hydrogen proposal. The calculations in this memo are similar to those in a memo by M. Bell & J.S. Bell, TH-3054-CERN and their appendix treats the $p+\bar{p} \to (p\bar{p})+\gamma$ radiative recombination, easily extended to the e^+e^- case of this memo.

They do treat the case of asymmetric temperatures which can be used here.

Further, the $kT \approx 1$ eV used may be an order of magnitude or two large, which improves the reaction rates. The beam areas and the accelerator circumference are all subject to an accelerator design. Finally the $1/\gamma^2$ dependence of the lab-frame rates suggests large gains as the energy decreases from 20 MeV (e.g. a factor of 4 at 10 MeV).

Finally, tricks to deal with the long-lived Ps states (such as spin-flipping from the triplet to the singlet states) by an electromagnetic process may increase the annihilation rates.